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PROCESS CAPABILITY ANALYSES BASED ON RANGE WITH TRIANGULAR FUZZY NUMBERS

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ABSTRACT

Process Capability Indices (PCI) is an effective and excellent method of measuring product quality and determine whether the production process produce product within the specified limits. C_p , C_{pk} , C_{pm} and C_{pmk} are most used traditional process capability indices. In this paper, the fuzzy set theory concept is used to extent the process capability analyses by using the range (estimate of σ) and fuzzy midrange transform techniques to get more information and flexibility of the PCA system. The fuzzy tolerance control limits for $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ are developed and compared their performances with a numerical example. The result shows that the $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ control limits are the evidence of improvement in the process performance.

KEYWORDS: Fuzzy Range Control Limits, Fuzzy Tolerance Limits, Process Performance Fuzzy Limits, Process Fuzzy Control Limits, Triangular Fuzzy Limits

INTRODUCTION

Process capability indices are widely used to check whether the production process performance is within the customer's requirement. A process meeting customer requirement is called "capable". A process capability (PCI) is a process characteristic relative to specifications. The setting and communication becomes vey simpler and easier by using process capability indices to express process capability between manufacturers ad customers. The use of these indices provides a unit less language for evaluating not only the actual performance of production processes, but the potential performance as well.

The indices are intended to provide a concise summary of importance that is readily usable. Engineers, manufacturers and suppliers can communicate with this unit less language in an effective manner to maintain high process capabilities and enable cost savings. The capability of a process and effectiveness of control charts are directly related. PCI's have become very popular in assessing the capability of manufacturing process in practice during the past decade. Process capability can be broadly defined as the ability of a process to meet customers' expectations which are defined as specification limits.

The PCA's compares the output of a process to the specification limits by using the process capability indices (PCI). In the review of the literature, C_p , C_{pk} , C_{pm} and C_{pmk} are some of indices used to identify. In this paper, the study is structured in the following order. The literature reviews of various Process capability Indices were discussed in the first section. Secondly, the approximate tolerance limits for $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ based on range are constructed by using chi-square distributions by Patnaik approximation. Thirdly, the approximate tolerance limits for $\ddot{C}_p$ based on range is transformed to fuzzy Triangular number tolerance limits for $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ based on range by using α-level fuzzy midrange transform technique. Fourthly, the fuzzy estimations of $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$chart based on range by using α-cuts are constructed. Next, the applications to understanding the fuzzy estimations of $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ chart based on range by using α-cuts are given and finally, the conclusions are presented.

LITERATURE REVIEWS

There are different indices that are given in literature review. In this case the quality characteristics X and the corresponding random sample (x_1, x_2,x_3,……x_n) are normal, in fact $X \sim N(\mu, \sigma^2)$. Let LSL and USL denotes the lower and upper specification limits, $M = \frac{(\text{USL} + \text{LSL})}{2}$, the midpoint of tolerance interval (LSL, USL), t the target value for μ, which we assume that t = m.

Kane (1986) was suggested the simplest and process potential index defined as

$$C_p = \frac{USL-LSL}{6\sigma} \tag{1}$$

This index is a ratio of tolerance region to process region. It is clear that this method consider the variation of process. 6 σ in denominator of above fraction are based on assumption of approximately normal of data. It will react to change in process dispersion but not change of process location.

Sullivan (1984) has been suggested a new process index C_{PK} in order to reflect departure from the target value as well as change in the process variation, is given by

$$C_{pk} = \frac{Min\,(USL-\mu,\ \ \mu-LSL)}{3\sigma} \tag{2}$$

Chan, Chen and Springe (1988) given another new process index CPM, in order to be a sign of departure from the target value as well as change in the process variation, defined and is given by

$$C_{pm} = \frac{USL-LSL}{6\sqrt{\sigma^2+(\mu-t)^2}} \tag{3}$$

6σ, 3σ in denominator of above fractions are based on assumption of approximately normal of data. In the other words main constraints in above indices is its normality assumptions. Departure from the target value carry more weight with the other well known capability indices is defined by

$$C_{pmk} = min\left\{\frac{USL-\mu,}{3\sqrt{\sigma^2+(\mu-t)^2}}, \frac{\mu-LSL}{3\sqrt{\sigma^2+(\mu-t)^2}}\right\} \tag{4}$$

C_{pm} and C_{pmk} react more both in dispersion and location than C_{pk}. C_{pmk} is more sensitive than $\mathbf{C_{pm}}$ to deviations from the target value T.

Kerstin Vannman (1995), has been given the unified approach. By varying the parameters of this class, we can find indices with different desirable properties. The proposed new, indices depend on two non-negative parameters, u and v, as

$$C_p(u,v) = \frac{d-u|\mu-M|}{3\sqrt{\sigma^2+v(\mu-T)^2}} \tag{5}$$

It is easy to verify that: $C_p(0,0) = C_{p;}$ $C_p(1,0) = C_{pk}$ $C_p(0,1) = C_{pm;}$ $C_p(1,1) = C_{pmk;}$

Form the study of $C_p(u,v)$, large value of u and v will make the index $C_p(u,v)$ more sensitive to departure from the target value. A slight modification gives general index class which includes

$$C_p(u_1,u_2,v) = \frac{d-u_1|\mu-M|-u_2|T-M|}{3\sqrt{\sigma^2+v(\mu-T)^2}} \tag{6}$$

$$C_p(0,1,1) = C_{pm}^*$$

The five $C_p,\ C_{pk},\ C_{pm},\ C_{pmk}\ and\ C_{pm}^*$ are equal when μ = T =M, but differ when $\mu \neq T$.

Carr (1991), The other approach is defined to use non conforming ratios as an index for capability process for the first time

$$NC = p = p\{x \notin [L,U]\} = 1 - \{\varphi\left(\frac{U-\mu}{\sigma}\right) - \varphi\left(\frac{L-\mu}{\sigma}\right)\} \tag{7}$$

This approach based on the stepwise loss function as below:

$$loss = \begin{cases} 0 & L \leq X \leq U \\ 1 & otherwisw \end{cases}$$

Clements (1989), in an influential paper, suggested that "6σ" be replace by the length of the interval between the upper and lower 0.135 percentage points of the distribution of X and defined

$$C'_p = \frac{U-L}{(\varepsilon_{1-a}-\varepsilon_a)} \tag{8}$$

Greenwich and Jahr-Schaffrath (1995) defined the index C_{pp} which provides an uncontaminated separation between information concerning process accuracy and process precision as follows

$$C_{pp} = C_{ia} + C_{ip} \tag{9}$$

Where the inaccuracy index $C_{ia} = (\frac{\mu-T}{\sigma})^2$ and

Imprecision index $C_{ip} = (\frac{\sigma}{D})^2$ and $D = \min(T - LSL, USL - T)/3$.

Bernardo and Irony (1996), The bayes capability index $C_{B(D)}$ given by

$$C_{B(D)} = \frac{1}{v}\phi^{-1}\{\Pr(y \in A/D)\} \tag{10}$$

A Bayesian index is proposed to evaluate process capability which within a decision –theoretical framework, directly assesses the proportion of future work may be expected to lie outside the tolerance limits. Hsin-Lin Kuo (2010) extended the capability indices by getting approximate tolerance limits for **Cp** capability chart based on Range

Approximate Tolerance Limits for $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ & $\tilde{C}_{pmk}$ Based on Range using χ^2 Distribution

Let $X_1, X_2, X_3,\ldots\ldots, X_n$ be a random sample of size n drawn from a normal population with mean μ and standard deviation σ. The mean of the m ranges will be denoted by $\bar{R}_{m,n}$ and the range of a single sample of size n is denoted by $R_{1,n}$.

When E $(\bar{R}_{m,n}) = \sigma d_2$ and Var(R) = $\sigma^2 d_3^2$, the mean and variance of $\bar{R}_{m,n}/\sigma$ are given by

$$E(\bar{R}_{m,n}/\sigma) = E(R_{1,n}) = d_2 \quad \text{and}$$

$Var(\bar{R}_{m,n}/\sigma) = Var(R_{1,n}/\sigma) = d_3^2/m$, respectively

Then $\bar{R}_{m,n}/d_2$ is the unbiased estimator of σ, where d_2 and d_3 are constants. According to Patnaik, it has been shown that $\bar{R}_{m,n}/\sigma$ is approximately distributed as $\frac{c\sqrt{\chi^2}}{\sqrt{v}}$. That is

$$(\bar{R}_{m,n}/\sigma)^2 \equiv C^2 \frac{\chi_v^2}{v} \quad \text{and} \quad (\bar{R}_{m,n}/\sigma)^2 \text{ X } \frac{v}{C^2} \equiv \chi_v^2$$

Where χ_v^2 denotes a chi-square distribution with v degrees of freedom, and c and v are constants which are functions of the first two moments of the range variable, given by

$$v = \frac{1}{(-2 + 2\sqrt{1 + 2(d_3/d_2)^2/m}}$$

$$c = d_2 \text{ x } \sqrt{v/2} \text{ x} \Gamma(v/2) / \Gamma((v+1)/2) \approx d_2 (1 + 1/(4v))$$

Using this relations, the values of c and v can be easily obtained for any n and m. Assume that the process the measurement follows N (μ, σ^2), the normal distribution, the $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ index are given below

$$\breve{C}_{pk} = \frac{Min(USL-\mu,\ \mu-LSL)}{3\breve{\sigma}} \tag{11}$$

$$\breve{C}_{pm} = \frac{USL-LSL}{6\sqrt{\breve{\sigma}^2+(\mu-t)^2}} \tag{12}$$

$$\breve{C}_{pmk} = min\left\{\frac{USL-\mu,}{3\sqrt{\breve{\sigma}^2+(\mu-t)^2}}, \frac{\mu-LSL}{3\sqrt{\breve{\sigma}^2+(\mu-t)^2}}\right\} \tag{13}$$

Apply a simple approximation procedure based on range we can obtain the tolerance limits of the estimator of $\ddot{C}_p$. The 100(1 – α) approximate tolerance limits for $\ddot{C}_p$ together with R charts

$$1 - \alpha = P\left(\chi^2_{1-\frac{\alpha}{2}} \leq \chi^2_R \leq \chi^2_{\frac{\alpha}{2}}\right) = P\left(J_1\widehat{C_p} \leq \widehat{C_p} \leq J_2\widehat{C_p}\right)$$

Where $J_1 = \frac{d_2}{c} \text{ x } \sqrt{\frac{v}{\chi^2_{\alpha/2}}}$ and $J_2 = \frac{d_2}{c} \text{ x } \sqrt{\frac{v}{\chi^2_{1-\alpha/2}}}$ and $\chi^2_{\alpha/2}(v)$ is the upper $\alpha/2$ quantile of the chi-square distribution with v degrees of freedom. So, the 100(1 – α) approximate tolerance limits for $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ based on range is given below

Upper Tolerance Limit			**Center Line**			**Lower Tolerance Limit**			
$J_2\tilde{C}_{pk}$	$J_2\tilde{C}_{pm}$	$J_2\tilde{C}_{pk}$	$\tilde{C}_{pk}$	$\tilde{C}_{pm}$	$\tilde{C}_{pmk}$	$J_1\tilde{C}_{pk}$	$J_1\tilde{C}_{pm}$	$J_1\tilde{C}_{pmk}$	(14)

Fuzzy Transformation Techniques and α - Level Fuzzy Mid – Range

In this study, α- level fuzzy mid – range transformation techniques used for the construction of approximate tolerance limits for $\ddot{C}_p$ based on range to fuzzy triangular number control chart. The α- level fuzzy mid – range f^{α}_{mr} is defined as the midpoint of the ends of the α- cuts. An α- level cut, denoted by A^{α} , is a nonfuzzy set that comprises ale element whose membership is greater than or equal to α. If a^{α} and b^{α} are the end points of A^{α} , then

$$f_{mr}^{\alpha} = \frac{a^{\alpha}+b^{\alpha}}{2} \tag{15}$$

α- level fuzzy mid – range of the sample is given by,

$$s_{Cpk}^{\alpha} = \frac{\ddot{C}_{pkaj}+\ddot{C}_{pkcj} \ +\alpha[(\ddot{C}_{pkbj}-\ddot{C}_{pkaj})-(\ddot{C}_{pkc}-\ddot{C}_{pkbj})]}{2} \tag{16}$$

Fuzzy Approximate Tolerance Limits for $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ Based on Range

α –cut Fuzzy tolerance limits for $\ddot{C}_{pk}$ based on Range

Upper tolerance limit: $\{J_2\hat{C}_{pka}^{\alpha} , J_2\hat{C}_{pkb} , J_2\hat{C}_{pkc}^{\alpha}\}$

Center line: $$\left\{\hat{C}_{pka}^{\alpha} = \frac{Min\ (USL-\mu,\ \mu-LSL)}{6\left(\bar{R}_a^{\alpha}/d_2\right)} , \hat{C}_{pkb}^{\alpha} = \frac{Min\ (USL-\mu,\ \mu-LSL)}{6\left(\bar{R}_b/d_2\right)} , \hat{C}_{pkc}^{\alpha} = \frac{Min\ (USL-\mu,\ \mu-LSL)}{6\left(\bar{R}_c^{\alpha}/d_2\right)}\right\} \tag{17}$$

Lower tolerance limit: $\{J_1\hat{C}_{pka}^{\alpha} , J_1\hat{C}_{pkb} , J_1\hat{C}_{pkc}^{\alpha}\}$

Where $$\hat{C}_{pka}^{\alpha} = \hat{C}_{pka} + \alpha(\hat{C}_{pkb} - \hat{C}_{pka}) \tag{18}$$

$$\hat{C}_{pkc}^{\alpha} = \hat{C}_{pkc} + \alpha(\hat{C}_{pkc} - \hat{C}_{pkb}) \tag{19}$$

α- level Fuzzy midrange tolerance limits for $\ddot{C}_{pk}$ based on Range

Upper tolerance limit: $\left\{\frac{J_2 C_{pkaMR}^{\alpha}+ J_2 C_{pkcMR}^{\alpha}}{2}\right\}$

Center line: $$\left\{C_{pkMR}^{\alpha} = \frac{C_{pka}^{\alpha}+C_{pkc}^{\alpha}}{2}\right\} \tag{20}$$

Lower tolerance limit: $\left\{\frac{J_1 C_{pkaMR}^{\alpha}+ J_1 C_{pkcMR}^{\alpha}}{2}\right\}$

The condition of the process control can be defined by

$$\textbf{process control} = \begin{cases} \text{incontrol} & J_1 C_{pkMR}^{\alpha} \le s_{CpkMR}^{\alpha} \le J_2 C_{pkMR}^{\alpha} \\ \text{out}-\text{of control} & \text{otherwise} \end{cases} \tag{21}$$

α –Cut Fuzzy Tolerance Limits for $\ddot{C}_{pm}$ Based on Range

α **–cut f**uzzy tolerance limits for $\ddot{C}_{pk}$based on Range can be calculated as follows.

Upper tolerance limit: $\{J_2\ddot{C}_{pma}^{\alpha} , J_2\ddot{C}_{pmb} , J_2\ddot{C}_{pmc}^{\alpha}\}$

Center line: $$\left\{\hat{C}_{pma}^{\alpha} = \frac{USL-LSL}{6\sqrt{\left(\bar{R}_a^{\alpha}/d_2\right)^2+(\bar{X}-t)^2}} , \hat{C}_{pmb} = \frac{USL-LSL}{6\sqrt{\left(\bar{R}_b/d_2\right)^2+(\bar{X}-t)^2}} , \hat{C}_{pmc}^{\alpha} = \frac{USL-LSL}{6\sqrt{\left(\bar{R}_c^{\alpha}/d_2\right)^2+(\bar{X}-t)^2}}\right\} \tag{22}$$

Lower tolerance limit: $\{J_1\ddot{C}_{pma}^{\alpha} , J_1\ddot{C}_{pmb} , J_1\ddot{C}_{pmc}^{\alpha}\}$

Where $\ddot{C}^{\alpha}_{pma} = \ddot{C}_{pma} + \alpha(\ddot{C}_{pmb} - \ddot{C}_{pma})$ (23)

$$\ddot{C}^{\alpha}_{pmc} = \ddot{C}_{pmc} + \alpha(\ddot{C}_{pmc} - \ddot{C}_{pmb}) \quad (24)$$

α- level Fuzzy midrange tolerance limits for $\ddot{C}_{pm}$ based on Range

Upper tolerance limit: $\left\{\frac{J_2 C^{u}_{pmaMR} + J_2 C^{u}_{pmcMR}}{2}\right\}$

Center line: $\left\{C^{\alpha}_{pmMR} = \frac{\ddot{C}^{\alpha}_{pma} + \ddot{C}^{\alpha}_{pmc}}{2}\right\}$ (25)

Lower tolerance limit: $\left\{\frac{J_1 C^{u}_{pmaMR} + J_1 C^{u}_{pmcMR}}{2}\right\}$

The condition of the process control can be defined by

$$\textbf{process control} = \begin{cases} \text{incontrol} & J_1 C^{\alpha}_{pmMR} \le s^{\alpha}_{CpmMR} \le J_2 C^{\alpha}_{pmMR} \\ \text{out} - \text{of control} & \text{otherwise} \end{cases} \quad (26)$$

α –cut Fuzzy tolerance limits for $\ddot{C}_{pmk}$ based on Range

Upper tolerance limit: $\{J_2 \hat{C}^{\alpha}_{pmka}, J_2 \hat{C}_{pmkb}, J_2 \hat{C}^{\alpha}_{pmkc},\}$

Center line: $\left\{\hat{C}^{\alpha}_{pmka} = \frac{Min\,(USL-\mu,\ \mu-LSL)}{3\sqrt{\left(\hat{R}^{\alpha}_{a}/d_2\right)^2 + (\bar{X}-t)^2}}, \check{C}_{pmkb} = \frac{Min\,(USL-\mu,\ \mu-LSL)}{3\sqrt{\left(\bar{R}_b/d_2\right)^2 + (\bar{X}-t)^2}}, \hat{C}^{\alpha}_{pmkc} = \frac{Min\,(USL-\mu,\ \mu-LSL)}{6\sqrt{\left(\hat{R}^{\alpha}_{c}/d_2\right)^2 + (\bar{X}-t)^2}}\right\}$ (27)

Lower tolerance limit: $\{J_1 \hat{C}^{\alpha}_{pmka}, J_1 \hat{C}_{pmkb}, J_1 \hat{C}^{\alpha}_{pmkc},\}$

Where $\hat{C}^{\alpha}_{pmka} = \hat{C}_{pmka} + \alpha(\hat{C}_{pmkb} - \hat{C}_{pmka})$ (28)

$$\hat{C}^{\alpha}_{pmkc} = \hat{C}_{pmkc} + \alpha(\hat{C}_{pmkc} - \hat{C}_{pmkb}) \quad (29)$$

α- level Fuzzy midrange tolerance limits for $\ddot{C}_{pm}$ based on Range

Upper tolerance limit: $\left\{\frac{J_2 C^{u}_{pmkaMR} + J_2 C^{u}_{pmkcMR}}{2}\right\}$

Center line: $\left\{C^{\alpha}_{pmkMR} = \frac{\ddot{C}^{\alpha}_{pma} + \ddot{C}^{\alpha}_{pmc}}{2}\right\}$ (30)

Lower tolerance limit: $\left\{\frac{J_1 C^{u}_{pmakMR} + J_1 C^{u}_{pmckMR}}{2}\right\}$

The condition of the process control can be defined by

$$\textbf{process control} = \begin{cases} \text{incontrol} & J_1 C^{\alpha}_{pmkMR} \le s^{\alpha}_{CpmkMR} \le J_2 C^{\alpha}_{pmkMR} \\ \text{out} - \text{of control} & \text{otherwise} \end{cases} \quad (31)$$

Table 1: The Triangular Fuzzy Measurement Values & the Fuzzy Ranges

Sample	X_a					X_b					X_c				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	5.71	5.5	5.43	5.2	5.51	5.73	5.57	5.45	5.25	5.53	5.75	5.6	5.46	5.27	5.55
2	5.41	5.52	5.25	5.51	5.65	5.43	5.57	5.29	5.53	5.69	5.44	5.58	5.3	5.56	5.7
3	5.25	5.51	5	5.2	5.31	5.29	5.53	5.13	5.25	5.33	5.32	5.54	5.17	5.28	5.37
4	5.42	5.26	5.42	5.49	5.6	5.51	5.31	5.44	5.55	5.64	5.62	5.38	5.46	5.58	5.71
5	5.19	5.18	5.25	5.21	5.52	5.3	5.2	5.28	5.27	5.57	5.32	5.31	5.32	5.3	5.6
6	5.36	5.31	5.18	5.38	5.26	5.42	5.4	5.23	5.48	5.33	5.63	5.55	5.31	5.47	5.42
7	5.26	5.53	5.41	5.28	5.19	5.27	5.57	5.46	5.29	5.26	5.31	5.62	5.49	5.32	5.3
8	5.43	5.28	5.44	5.5	5.58	5.52	5.33	5.47	5.56	5.62	5.63	5.4	5.5	5.6	5.69
9	5.69	5.45	5.32	5.19	5.45	5.72	5.49	5.35	5.22	5.48	5.75	5.56	5.41	5.28	5.51
10	5.31	5.26	5.14	5.4	5.31	5.35	5.29	5.21	5.43	5.36	5.42	5.32	5.26	5.46	5.38
11	5.28	5.5	5.41	5.31	5.35	5.32	5.56	5.46	5.34	5.41	5.36	5.62	5.48	5.38	5.44
12	5.43	5.22	5.15	5.34	5.48	5.46	5.26	5.18	5.36	5.52	5.48	5.28	5.21	5.41	5.56
13	5.46	5.35	5.35	5.22	5.28	5.52	5.39	5.42	5.28	5.32	5.56	5.43	5.48	5.34	5.36
14	5.41	5.36	5.52	5.51	5.38	5.44	5.4	5.56	5.54	5.42	5.48	5.43	5.62	5.62	5.46
15	5.62	5.48	5.42	5.18	5.41	5.64	5.55	5.46	5.23	5.46	5.68	5.59	5.49	5.26	5.49

Applications of α - Level Fuzzy Midrange Tolerance Limits for $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$

An application was given by Sentruk and Erginel [9] made on controlling piston inner diameters in compressors. The same data have been considered with the first fifteen samples each with size 5 (the total measurements is 5 X 15 = 75) were taken from the production process by different operators. Quality experts evaluated each value by a fuzzy number because the variability of a measurement system includes operators and gauge variability. For example, the numeric observed value 5.73 can be measured by various operators between 5.71 and 5.76.The upper and lower specification limits of the process are defined as approximately 5.7 and approximately 5.1, respectively. These measurements are converted into triangular fuzzy numbers and given in Table 1. Fuzzy control limits are calculated according to the procedures given in the previous sections.

From the process data, If we assume that mean $(\bar{x})$ and $\bar{R}_{m,n}/d_2$ is the estimate of μ and σ.

We obtain the following result:

$\overline{\mathrm{R}} = 0.332$ $d_2 = 2.326$ $d_3 = 0.8641$, c = 2.3367 v = 54.59

Target value t = 5.4 Mean $(\bar{x})$ = 5.4163

Table 2: α - Level Fuzzy Midrange Tolerance Limits for $\ddot{C}_{pk}$ Based on Range

Sample No.	$\ddot{C}_{pka}$	$\ddot{C}_{pkb}$	$\ddot{C}_{pkc}$	s^{α}_{CpkMRj}	$0.5357 \leq s^{\alpha}_{CpkMRj} \leq 1.1546$
1	0.3497	0.3134	0.2811	0.3141	Out of control
2	0.4497	0.3838	0.3567	0.3906	Out of control
3	0.2341	0.3993	0.4945	0.3871	Out of control
4	0.5975	0.4934	0.3524	0.4869	Out of control
5	0.3877	0.4694	0.6978	0.4951	Out of control
6	0.7676	0.8436	0.5427	0.7776	In control
7	0.5336	0.6753	0.7075	0.6561	In control
8	0.6564	0.5347	0.3636	0.5261	Out of control
9	0.4342	0.3846	0.3266	0.3831	Out of control
10	0.5487	0.8035	1.0389	0.8001	In control
11	0.9515	0.9110	0.7276	0.8860	In control
12	0.5263	0.5838	0.6380	0.5832	In control
13	0.7495	0.9239	0.9374	0.8958	In control
14	1.2793	1.1049	0.7264	1.0691	In control
15	0.4899	0.4387	0.3655	0.4349	Out of control

α - Level Fuzzy Midrange Tolerance Limits for $\ddot{C}_{pk}$ Based on Range

Upper tolerance limit $\{1.1546\}$

Center line: $\{0.6407\}$ (32)

Lower tolerance limit $\{0.5357\}$

s^{α}_{CpkMRj} have been calculated for all the 15 samples with respect to different operators by using equations (16) and are given in the table 2. As shown in the table 2, the process capability index decreased and increased in samples like 1, 2, 3, 4, 5, 8, 9 and 15 which indicating the process is out of control and need of investigations. Out of 15 samples 8 sample points lies outside of the process control.

Table 3: α - Level Fuzzy Midrange Tolerance Limits for $\ddot{C}_{pm}$ Based on Range

Sample No.	$\ddot{C}_{pma}$	$\ddot{C}_{pmb}$	$\ddot{C}_{pmc}$	s^{α}_{CpmMRj}	$0.5816 \leq s^{\alpha}_{CpmMRj} \leq 1.2536$
1	0.4345	0.431	0.4136	0.4286	Out of control
2	0.5408	0.5001	0.4821	0.5041	Out of control
3	0.3796	0.5102	0.5832	0.5002	Out of control
4	0.6621	0.5952	0.4843	0.5875	In control
5	0.5112	0.5672	0.7552	0.5903	In control
6	0.7496	0.9003	0.6362	0.8277	In control
7	0.6235	0.732	0.7256	0.7119	In control
8	0.7303	0.6257	0.4854	0.6194	In control
9	0.4632	0.4522	0.4418	0.4523	Out of control
10	0.6208	0.8413	1.0900	0.8462	In control
11	1.0078	0.9547	0.7999	0.9369	In control
12	0.6213	0.6551	0.6625	0.6505	In control
13	0.8092	0.9604	0.9949	0.9400	In control
14	1.2880	1.0042	0.6811	0.9974	In control
15	0.5251	0.5293	0.4822	0.5203	Out of control

α - Level Fuzzy Midrange Tolerance Limits for $\ddot{C}_{pm}$ Based on Range

α- level Fuzzy midrange tolerance limits for $\ddot{C}_{pm}$based on Range calculated as

$$\begin{array}{ll} \text{Upper tolerance limit} & \{1.2536\} \\ \text{Center line:} & \{0.6874\} \\ \text{Lower tolerance limit} & \{0.5816\} \end{array} \quad (33)$$

$s^{\alpha}_{Cpm\,MRj}$ have been calculated for all the 15 samples with respect to different operators by using equations (16) and are given in the table 3. As shown in the table 3, the process is in control but the process capability index decreased and increased in samples like 1, 2,3, 9 and 15 which indicating the process is out of control and need of investigations. Out of 15 sample only 5 samples lies outside of the process control. The α - level Fuzzy midrange tolerance limits for $\ddot{C}_{pm}$based on Range can be used to control the process.

Table 4: α - Level Fuzzy Midrange Tolerance Limits for $\ddot{C}_{pmk}$ Based on Range

Sample No.	$\ddot{C}_{pmka}$	$\ddot{C}_{pmkb}$	$\ddot{C}_{pmkc}$	s^{α}_{CpmMRj}	$0.5246 \leq s^{\alpha}_{CpmMRj} \leq 1.1308$
1	0.5064	0.4502	0.3875	0.4491	Out of control
2	0.6633	0.6398	0.5731	0.6323	In control
3	0.1949	0.3504	0.4588	0.3421	Out of control
4	0.7460	0.7737	0.5690	0.7331	In control
5	0.2897	0.4235	0.6797	0.4449	Out of control
6	0.4947	0.8163	0.7974	0.7567	In control
7	0.4863	0.6588	0.7450	0.6437	In control
8	0.8423	0.8342	0.5883	0.7926	In control
9	0.4941	0.5305	0.4810	0.5155	Out of control
10	0.3807	0.6394	0.9737	0.6526	In control
11	0.9070	1.012	0.9492	0.9827	In control
12	0.4639	0.559	0.6360	0.5558	In control
13	0.6258	0.9155	1.1077	0.8985	In control
14	1.4426	1.2453	0.9581	1.2295	Out of control
15	0.5636	0.6493	0.5875	0.6235	Out of control

α - Level Fuzzy Midrange Tolerance Limits for $\ddot{C}_{pm}$ Based on Range

α- level Fuzzy midrange tolerance limits for $\ddot{C}_{pm}$based on Range calculated as

$$\begin{array}{ll} \text{Upper tolerance limit} & \{1.1308\} \\ \text{Center line:} & \{0.6874\} \\ \text{Lower tolerance limit} & \{0.5246\} \end{array} \quad (34)$$

s^{α}_{CpmMRj} have been calculated for all the 15 samples with respect to different operators by using equations (16) and are given in the table 4.

As shown in the table4, the process is in control but the process capability index decreased and increased in samples like 1, 3, 5, 9, 14 and 15 which indicating the process is out of control and need of investigations. Out of 15 sample only 6 samples lies outside of the process control. The α - level Fuzzy midrange tolerance limits for $\ddot{C}_p$ based on Range can be used to control the process.

CONCLUSIONS

In this paper, it is proposed a new methodology of constructing the fuzzy tolerance control limits for $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ in fuzzy set theory. Process capability analysis is an important tool to improve the process performance. The fuzzy tolerance control limits for $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$, improve the process control as compared with traditional control limits. By comparing the fuzzy control limits of $\tilde{C}_{pk}$, $\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$, the $\tilde{C}_{pm}$ fuzzy control limits shows that the out of control points is fewer than the $\tilde{C}_{pk}$ and $\tilde{C}_{pm}$.

The $\tilde{C}_{pm}$ fuzzy control limits is the enhanced process limit among the other fuzzy control limits in 3σ and also it shows the substantiation in improvement in the process performance. Fuzzy set theory is also befitted method for improving the process performance in process capability analysis. In the future, the performance of the process is assessing by using other transformation techniques in fuzzy logic.

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